Counter-current flow division for heat absorption from two isothermal parallel plates

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Introduction

The heat transfer between the fluid flow and two parallel plates is a benchmark, which frequently has been investigated [1]. This particular flow arrangement is the basic for many complicated heat exchangers containing flat plates as the walls of the flow channels. The simplified model for this particular heat transfer problem in which the axial conduction is negligible and flow regime is laminar, is known as the classical Graetz problem [2, 3]. For fluid flow with low Prandtl number, the axial conduction significantly affects the temperature distribution. When considering the axial conduction in the modeling of the flow between two heated parallel plates, it is known as extended Graetz problem [4-7].

The rate of heat transfer depends on the temperature difference between the flow and channel walls, the surface of heat transfer, and also convective heat transfer coefficient over the thermal boundaries [8]. For increasing the rate of the heat transfer in the presence of a fixed thermal boundary condition, one should increase the heat transfer surface and/or convective heat transfer coefficient. Convective heat transfer coefficient depends on the flow pattern through the parallel-plate channel. This coefficient was frequently measured in the research studies for different flow patterns [9-11].

In practice, the occupation space for installing a parallel-plate channel flow is restricted. In this case, the external surface of the channel is fixed and the only way for increasing the surface of heat transfer is to use the internally finned type channels [12, 13]. The main deficiency of the internally finned wall for parallel-plate channel is the deposition of the suspended particles on the fins, which gradually makes a blockage against the flow and also decreases thermal performance of the channel. Wavy channel wall is another solution for this purpose with less unfavorable deposition effect [14-16].

An alternative solution for improving thermal performance of a parallel-plate channel flow with restriction of the occupation space is to change the flow arrangement within the two channel walls. Ho et al [17-20] inserted a flat plate with negligible thickness between the walls of the parallel-plate channel to make two sub-channels. The incoming flow passes through the incoming sub-channel and then returns through the outgoing sub-channel, while both incoming and outgoing flows exchange the heat with each other as well as channel walls. They showed that doubling the flow passage within the parallel-plate channel increased the thermal performance. Goodarzi and Mazharmanesh [21] changed the flat separating plate in the double-pass parallel-plate channel with a sinusoidal one and showed that it improved the thermal performance a little. Ho et al [22,23] also studied the multi-pass flow arrangement within the parallel-plate channel. The results of these studies also indicated a more improvement compared to the double-pass arrangement, but with significantly greater power consumption for pumping the flow through the sub-channels.

The mass-flow rate within each sub-channel of the above mentioned multi-pass arrangements was the same as one was flow-
The basic problem is a laminar flow between two parallel plates. The temperature, pressure, and velocity profiles of the two parallel plates are the same and uniform. Figure 2(a) shows the geometrical dimensions of the basic flow arrangement, which is referred to as the single-pass flow arrangement. The ratio of the length to the width of the channel is 120. By symmetrically inserting two zero thickness plates between the main parallel plates, a new counter-current flow arrangement is constructed without changing the external dimensions. The total flow rate is divided to two streams one of them is supplied to the internal sub-channel and the other one is supplied to the external sub-channels. Accordingly, in the first scheme, which is referred as flow division regime 1, the total flow rate is equally divided to two streams one of them is supplied to the internal sub-channel and the other one is supplied to the external sub-channels. Consequently, for the second scheme, which is referred as flow division regime 2, the total flow rate is divided to two streams in the manner that each division is proportional to the cross section area of the corresponding sub-channel. According to Figure 2, $Q_1 = Q/2$. In the second scheme, which is referred as flow division regime 2, the total flow rate is divided to two streams in the manner that each division is proportional to the cross section area of the corresponding sub-channel. According to Figure 2 and Eq. (5), $Q_1 = KQ$. Therefore, the flow division regimes 1 and 2 are the same when $K = 0.5$.

**Governing equation and boundary conditions**

The width between the two parallel plates is so small in comparison to the plate length. Therefore, flow regime within the parallel plates is laminar and incompressible. The process of heat transfer from walls to the fluid flow is steady. The flows through the sub-channels are hydraulically

![Schematics of the present counter-current and two double-pass flow arrangements.](Image)

![Schematics of the single-pass (a) and the proposed counter-current (b) channels.](Image)
fully developed. Hence, the energy equation governs the thermal performance of the present particular problem completely. It is presented in the vector form as follows

$$\rho c_p \left( \overrightarrow{V} \cdot \nabla \right) T = k \nabla^2 T$$ \hspace{1cm} (6)

where, $\overrightarrow{V}$, $k$, and $c_p$ are velocity vector, thermal conductivity, and thermal capacity of the fluid, respectively.

The velocity profiles through the sub-channels are hydraulically fully developed corresponding to the volumetric flow rates. The flows enter the sub-channels with uniform temperature, i.e. $T_i$. Two main parallel plates have the same uniform temperature, i.e. $T_0$. There is no need for applying a particular thermal boundary condition on the impermeable separating plate, because it is a zero thickness boundary. It means the temperature and its gradient across the inserted plates are continues. Since the flow field and all hydraulic and thermal boundary conditions are symmetric around the centerline, it is better to simulate one-half of the flow field. So, all normal gradients of the dependent variables are set to zero on the symmetry plan.

**Numerical procedure**

In the present study the axial conduction is considered as well as transverse conduction. Although there may be an analytical solution for this particular problem, to avoid the singularity problem at the edge point and also mathematical complexities [25], a simple numerical finite volume method with structural grid system was used for finding the temperature distribution. A second order upwind scheme was used to discrete the differential terms in the governing equation. The resulted system of algebraic equation was implicitly solved using an iterative procedure until the temperature distribution reached to the numerically invariant values over the grid system.

To have the results independent of the grid numbers, several grid systems with different grid numbers were used for each case at the flow regime with greatest possible Graetz number, i.e. $G_z=1000$. The finalized grid system was selected when a distributed parameter such as Nusselt number on the wall did not dependent on the grid number.

The local Nusselt number is defined as

$$Nu = \frac{hW}{k} = \frac{W q''_w}{k(T_w - T_{in})}$$ \hspace{1cm} (7)

where, $q''_w$ and $T_{in}$ are heat flux on the channel wall and averaged temperature of the flow across the channel, respectively. Note that heat flux on the channel wall is proportional to the temperature gradient on the channel wall. To validate the accuracy of the applied numerical simulation, the outlet Nusselt number for two different thermal boundary conditions, i.e. uniform wall temperature and uniform heat flux, were computed in the single-pass arrangement at low Graetz number providing the thermally fully developed condition. The computed values for these particular cases are compared with the analytical ones [26] in Table 1. There are very good agreements between the numerically and analytically computed values. The differences between the numerical and analytical results are less than 1%.

**Results and discussions**

**Heat transfer enhancement**

A typical proposed flow arrangement with in the parallel-plate configuration is originally studied from heat transfer enhancement viewpoint. The best parameter for measuring the heat transfer enhancement is the local Nusselt number. It worth to mentioned that the entrances of the single-pass channel and sub-channel of the counter-current channel are allocated at $\xi=0$. Figure 3 shows the local Nusselt number along the wall of the single-pass and three typical proposed counter-current flow arrangements for two mentioned flow division regimes and different flow rates, i.e. different Graetz numbers. An overall inspection on this figure illustrates that the local Nusselt number of the counter-current channel varies from the entrance of the external sub-channel to the exit plane.
like the single-pass one. It is, the local Nusselt numbers are high at the channel entrances and then decrease gradually. The local Nusselt number of the counter-current channel is always greater than the single-pass one at the entrance of the external sub-channel regardless of the flow rate. Figure 3 shows that the local Nusselt number of the counter-current channel usually remains greater than the single-pass one after the entrance region, except for the low flow rates, namely Gz=10. The low flow rate will be investigated later with considering the temperature contours.

From geometrical viewpoint, Figure 3 illustrates that at high flow rate the local Nusselt number of the counter-current channel with greater cross section ratio is higher than the other ones regardless of the flow division regime. This conclusion deteriorates for low flow rate after some distance from entrance of the external sub-channel. For high flow rates, i.e. Gz>100, the local Nusselt number of the counter-current channel with K=0.7 and regime 1 of the flow division is greater than the other ones throughout the flow passage, while with K=0.3 and regime 2 of the flow division possesses the least local Nusselt number among the other ones. The differences among the values of the local Nusselt numbers corresponding to the three cross section ratios with the regime 2 of the flow division are less than those in the regime 1.

Now, some integrated parameters can be calculated for quantitative comparisons. The averaged Nusselt number is computed as

$$\bar{N_u} = \frac{1}{L} \int_0^L N_u dx$$  \hspace{1cm} (8)

Then, the enhanced heat transfer using the proposed counter-current flow arrangement is introduced as

$$E_Q = \frac{N_{u,c}-N_{u,s}}{N_{u,s}}$$  \hspace{1cm} (9)

where, indices c.c and s refer to counter-current and single-pass flow arrangements, respectively.

Figure 4 shows the averaged Nusselt numbers of all flow arrangements based on the Graetz number. Note that as was already mentioned the two flow division regimes for cross section ratio K=0.5 are the same. It helps us to compare the thermal performances of the other flow regimes and cross section ratios with this particular case and also single-pass arrangement. Considering Figure 4, it is clear that the averaged Nusselt number of the counter-current channel with cross section ratio K=0.7 and flow division regime 1 is always the best one among the other ones. Figure 5 shows the enhanced heat transfer when using the proposed counter-current flow arrangement. It declares that counter-current flow arrangements with cross section ratio K=0.7 provide the higher heat transfer enhancements, especially with flow division regime 1.

**Power consumption increment**

Changing the flow passage within the two parallel plates may provide the better thermal performance, but it may also increase the required power for supplying the same flow rate through the channel. Therefore, another index should be introduced from hydraulic viewpoint. The required power for pumping the prescribe volumetric flow rate within a typical channel is computed as

$$P = \sum \Delta p_i Q_i$$  \hspace{1cm} (10)

where, $\Delta p_i$ and $Q_i$ are the pressure drop and the corresponding volumetric flow rate through each sub-channel. Then, the power consumption increment is defined as

$$E_p = \frac{P_{c,c}-P_s}{P_s}$$  \hspace{1cm} (11)

The pressure drop through a typical parallel-plate channel with fully developed laminar flow is simply computed [26]. Table 2 lists the power consumption increment of the proposed counter-current flow arrangement compared to the two already mentioned double-pass ones. The listed values declare an important performance of the proposed counter-current flow arrangement; the power consumption increment of the proposed flow arrangement is significantly less than the two before investigated double-pass channels. It is favora-

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**Fig. 4.** Averaged Nusselt number versus the Graetz number for different channels, (left) flow division regime 1, (right) flow division regime 2.

**Fig. 5.** Heat transfer enhancement index of the counter-current channels at different flow rates.

**Fig. 6.** Overall performance index of the counter-current channels at different flow rates.
A new counter-current flow arrangement within the well-known parallel-plate channel has been proposed and investigated in the present study. Its thermal, hydraulically, and overall performances have been investigated compared to the basic single-pass arrangement. The obtained results illustrate that the proposed counter-current flow arrangement could improve the thermal performance. Also, the power consumption to supply the same flow rate into the counter-current flow arrangement increased, but the overall performance of the new counter-current flow arrangement was satisfactory for applications.

The most important characteristic of the proposed counter-current flow arrangement compared to the previously investigated double-pass arrangement is its less power consumption increment. Therefore, the proposed counter-current flow arrangement can be used as an alternative for multi-pass flow arrangement within the parallel-plate geometry with the less power requirement to enhance the heat transfer.

References

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